# Occlusive Components Analysis

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## Abstract

We study unsupervised learning in a probabilistic generative model for occlusion. The problem of occlusion is addressed from the perspective of multiple-causes models such as NMF [1], sparse coding [2], or ICA.

*The model features*

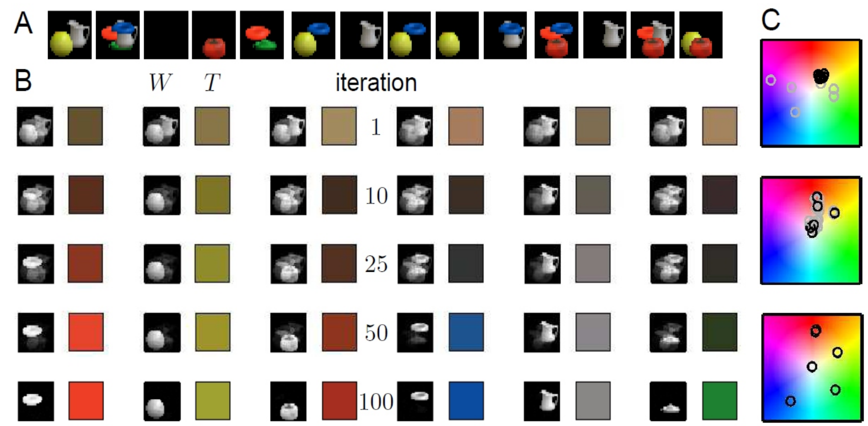
- binary hidden variables encoding object presence
- a vectorial hidden variable for object proximities
- a non-linear combination of objects based on their proximities
- two sets of parameters for each object: masks and features

*The learning algorithm*

- approximately maximizes the data likelihood
- is based on approximate EM (truncation approach, compare [5])
- is benchmarked on artificial and more realistic data

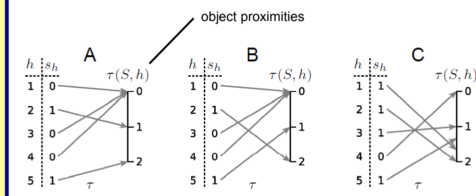
We find that the algorithm successfully extracts objects from images of clutter scenes. Future work aims at including transformation invariances (compare [3] and [4]) and gestalt laws.

## Numerical Experiments

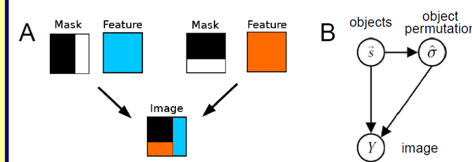



Application to images of cluttered objects. **A** Selection of 14 of the N = 500 data points. **B** Changes of the parameters *W* and *T* for the algorithm with *H* = 8 hidden units. Each row shows *W* and *T* for the specified EM iteration. **C** Feature vectors at different iterations stages displayed as point in color space. Black circles are the current model values and grey circles those of the previous iterations.

## Visualization of Permutation Mapping $\tau$




## Combining Masks and Features




**A** Illustration of how two object masks and features combine to generate an image (without noise).
**B** Graphical model of the generation process with hidden permutation variable $\hat{\sigma}$.

## The Generative Model

$$p(\vec{s}\,|\,\pi) = \prod_{h=1}^{H} \text{Bernoulli}(s_h;\pi) = \prod_{h=1}^{H} \pi^{s_h}\,(1-\pi)^{1-s_h}$$

$$p(\hat{\sigma}\,|\,\vec{s}) = \frac{1}{|\vec{s}|!} \quad \text{with} \quad \hat{\sigma} \in \mathcal{G}(|\vec{s}|)$$

$$\vec{T}_d(S,\Theta) = W_{h_o d}\,\vec{T}_{h_o}, \;\text{ where }\; h_o = \underset{h}{\text{argmax}}\{\tau(S,h)\,W_{hd}\},$$

$$\tau(S,h) = \begin{cases} 0 & \text{if } s_h = 0 \\ \frac{3}{2} & \text{if } s_h = 1 \text{ and } |\vec{s}| = 1 \\ \frac{\hat{\sigma}(h)-1}{|\vec{s}|-1} + 1 & \text{otherwise} \end{cases}$$

$$p(Y\,|\,S,\Theta) = \prod_{d=1}^{D} p(\vec{y}_d\,|\,\vec{T}_d(S,\Theta)), \quad p(\vec{y}\,|\,\vec{t}) = \mathcal{N}(\vec{y};\vec{t},\sigma^2\mathbb{1})$$

## The Learning Algorithm (EM-based)

E-Step:

$$\langle \mathcal{A}_{id}(S,W)\rangle_{q_n} \approx \frac{\sum_{S,(|\vec{s}|\leq\chi)} p(S,Y^{(n)}\,|\,\Theta')\,\mathcal{A}_{id}(S,W)}{\sum_{\tilde{S},(|\tilde{\vec{s}}|\leq\chi)} p(\tilde{S},Y^{(n)}\,|\,\Theta')} \qquad \text{with}$$

$$\mathcal{A}_{id}(S,W) := \lim_{\rho\to\infty} \mathcal{A}^{\rho}_{id}(S,W), \qquad \mathcal{A}^{\rho}_{id}(S,W) := \frac{(\tau(S,i)\,W_{id})^{\rho}}{\sum_{h=1}^{H}(\tau(S,h)\,W_{hd})^{\rho}}$$

M-Step:

$$W_{id} = \frac{\sum_n \langle \mathcal{A}_{id}(S,W)\rangle_{q_n}\,\vec{T}_i^T\,\vec{y}_d^{(n)}}{\sum_n \langle \mathcal{A}_{id}(S,W)\rangle_{q_n}\,\vec{T}_i^T\,\vec{T}_i}, \qquad \vec{T}_i = \frac{\sum_n\sum_d \langle \mathcal{A}_{id}(S,W)\rangle_{q_n}\,W_{id}\,\vec{y}_d^{(n)}}{\sum_n\sum_d \langle \mathcal{A}_{id}(S,W)\rangle_{q_n}\,(W_{id})^2}$$

## References

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