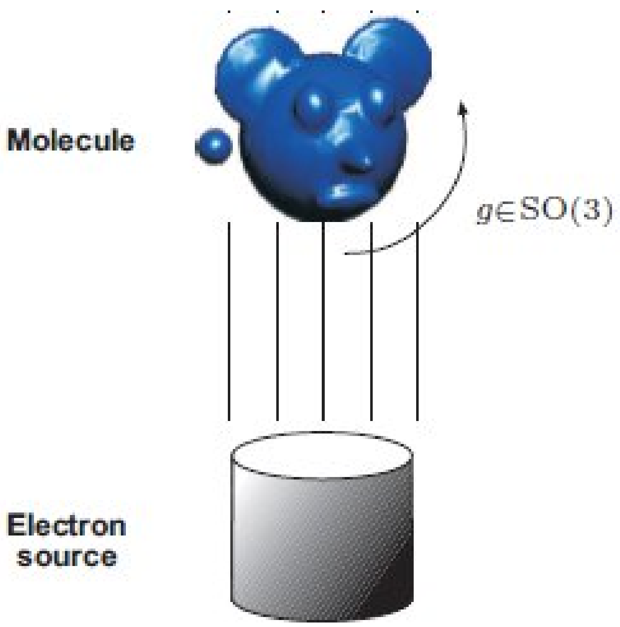
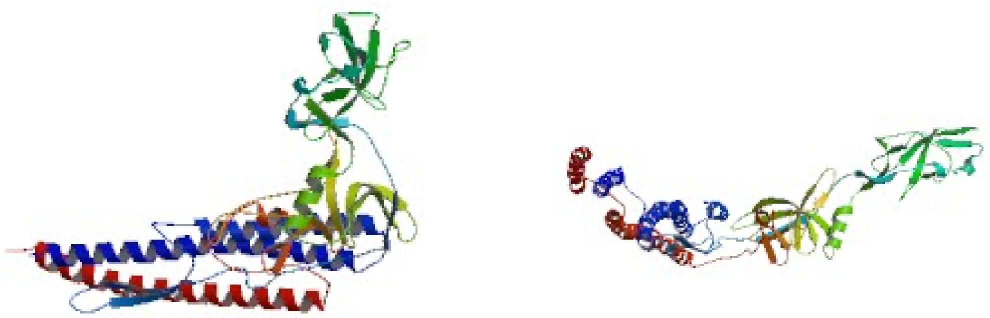
On Learning Rotations

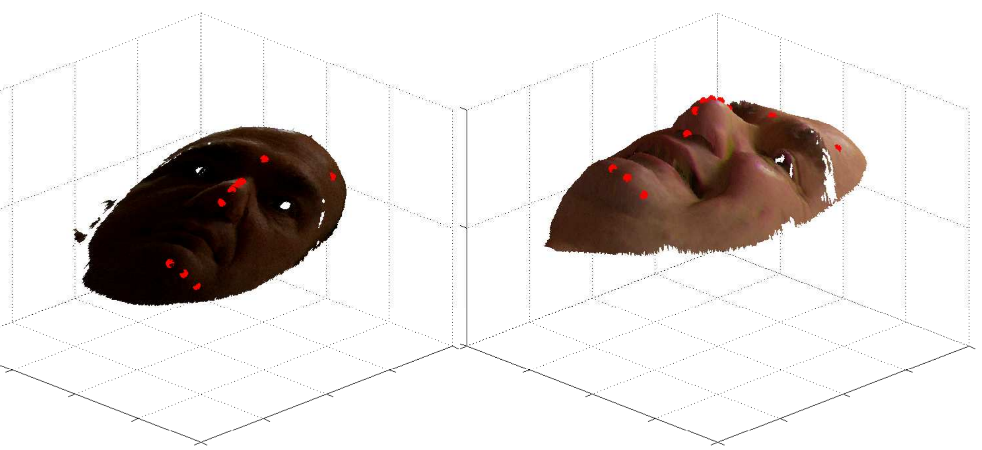
by Raman Arora

(http://www.cae.wisc.edu/~sethares/links/raman/LearnROT/vids.html)

Applications







Problem Statement and Solution

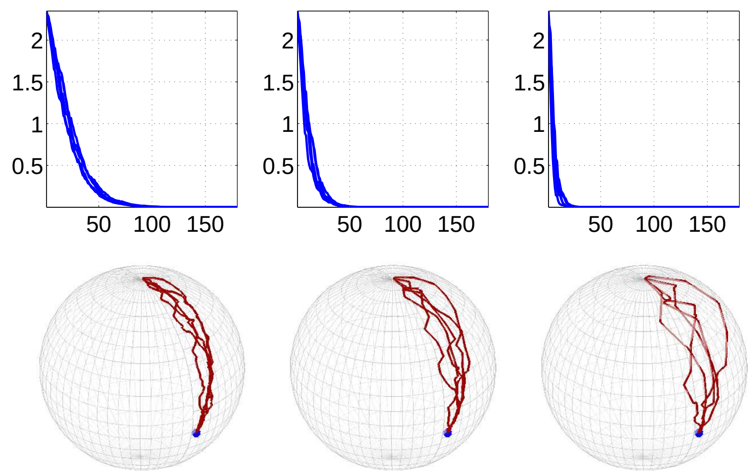
- Receive: a unit vector $\mathbf{x}_t \in \mathbb{R}^n$ at each instance.
- Predict: $\hat{\mathbf{y}}_t = \hat{\mathbf{R}}_t \mathbf{x}_t$ based on current estimate $\hat{\mathbf{R}}_t$ of $\mathbf{R}_*$.
- Receive: true rotated vector $\mathbf{y}_t = \mathbf{R}_* \mathbf{x}_t$
- Loss: Incur loss $L(\hat{\mathbf{y}}_t, \mathbf{y}_t)$.
- Update: the estimate $\hat{\mathbf{R}}_t$.

$$\hat{\mathbf{R}}_{t+1} = \arg\min_{\mathbf{R}} \quad \eta L_t(\mathbf{R}) + \Delta_F(\mathbf{R}, \hat{\mathbf{R}}_t)$$
$$\text{s.t.} \quad \mathbf{R}^T\mathbf{R} = \mathbf{I}, \quad \mathbf{R}\mathbf{R}^T = \mathbf{I}, \quad \det(\mathbf{R}) = 1$$

$\Delta_F(\mathbf{R}, \hat{\mathbf{R}}_t)$ - von Neumann divergence, $L_t = |\hat{\mathbf{y}}_t - \mathbf{y}_t|^2$

$$\hat{\mathbf{R}}_{t+1} = \hat{\mathbf{R}}_t \left(\mathbf{I} + \frac{\sin(\lambda)}{\lambda}\mathbf{S} + \frac{1-\cos(\lambda)}{\lambda^2}\mathbf{S}^2 \right)$$

where $\mathbf{S} = 2\eta \left(\hat{\mathbf{R}}_t^T \mathbf{y}_t \mathbf{x}_t^T - \mathbf{x}_t \mathbf{y}_t^T \hat{\mathbf{R}}_t \right)$, $\lambda = 2\eta\sqrt{1 - (\mathbf{y}_t^T \hat{\mathbf{y}}_t)^2}$.



Results