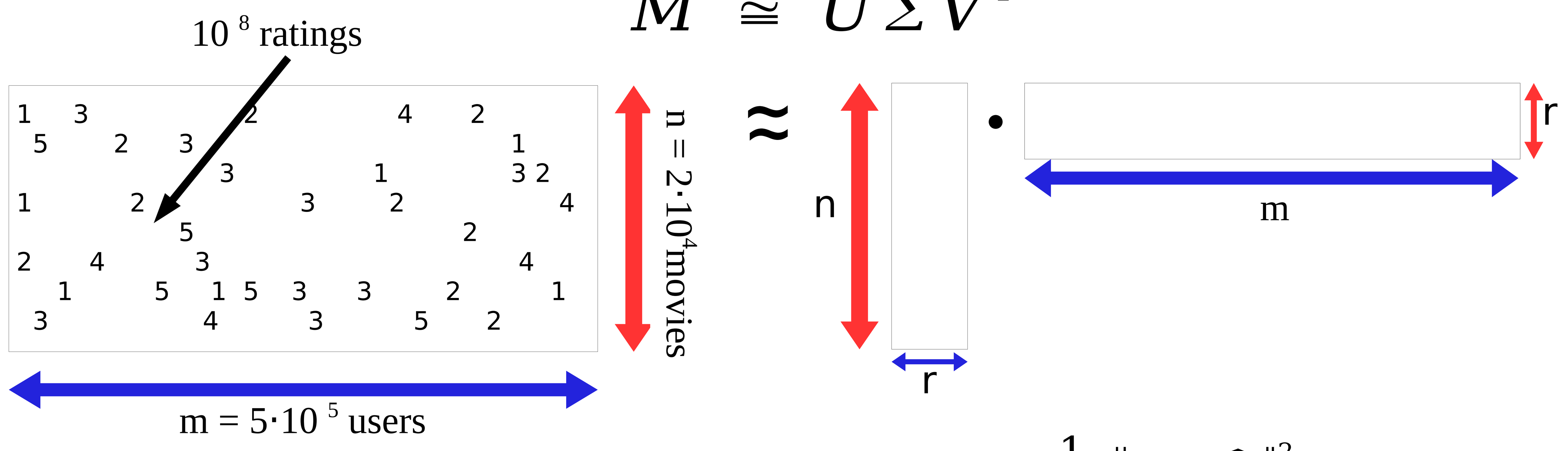
Matrix Completion from a Few Entries

Raghunandan H. Keshavan, Andrea Montanari, Sewoong Oh,
Department of Electrical Engineering, Stanford University

What is Matrix Completion?

$$M \cong U \Sigma V^T$$

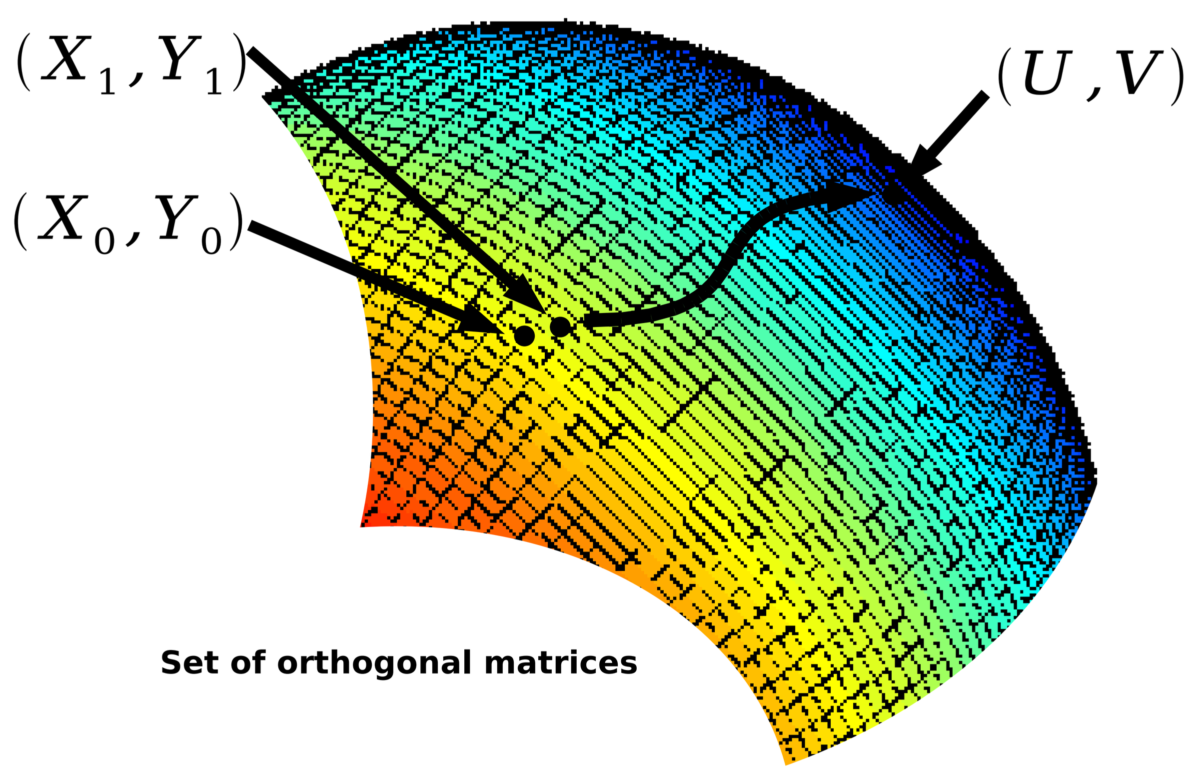


Problem : How many revealed entries |E| do we need to get $\frac{1}{mn}\|M-\widehat{M}\|_F^2 \le \delta$?

Algorithm

Algorithm [*OptSpace*]
Trim : Trim M^E to M^E;
Project : Project M^E onto Tr(M^E);
Clean : Minimize Cost F(X,Y), s.t. X,Y orthogonal.

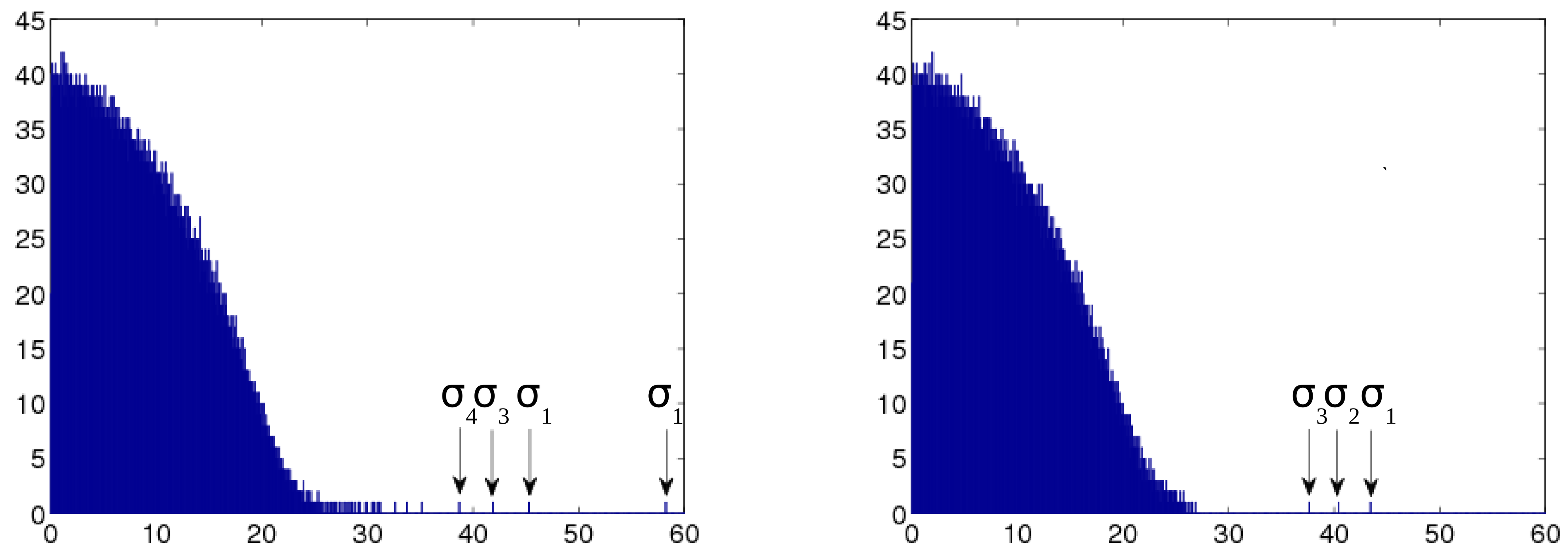
$$F(X,Y) = \min_{S} \sum_{i,j \in E} \left((XSY^T)_{ij} - M_{ij}\right)^2$$



SVD : $Tr(M^E) = \frac{mn}{|E|} \sum_{i=1}^{r} x_i \sigma_i y_i^T$

Problem : $\sigma_1 = \Omega(\log(n)/\log\log(n))$

Solution : Trimming



Histogram of singular values of a partially revealed random rank 3 matrix before(left) and after(right) trimming

Main Results

Theorem (Keshavan, Montanari, Oh, 2009 [1])

Assume $r = O(1)$, and let M be an $n\alpha \times n$ matrix satisfying (μ_0, μ_1)-incoherence with $\sigma_1(M)/\sigma_r(M) = O(1)$. If $|E| \ge C' n \log n$, then OptSpace *returns, whp., the matrix* M.

Theorem (Keshavan, Montanari, Oh, 2009 [2])

Let $N = M + Z$ with M as above and Z any $n\alpha \times n$ matrix. If $|E| \ge C' n \log n$, then (under appropriate technical conditions) OptSpace *with input N^E returns $\widehat{M}$ such that whp.,*

$$\frac{1}{\sqrt{mn}} ||M - \widehat{M}||_F \le C \frac{n\sqrt{\alpha r}}{|E|} ||Z^E||_2$$

Implementation

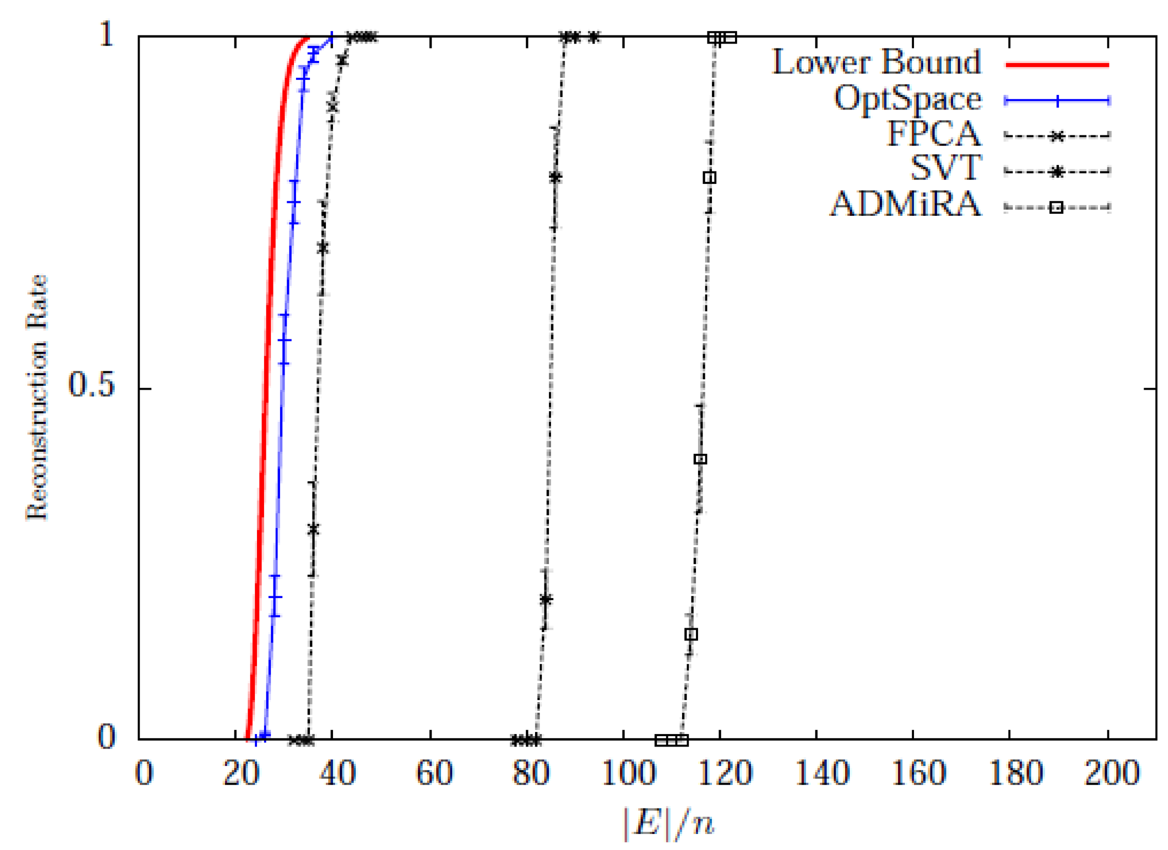


Figure 1: Reconstruction Rate vs. No. of samples revealed (m = n = 1000, r = 10)

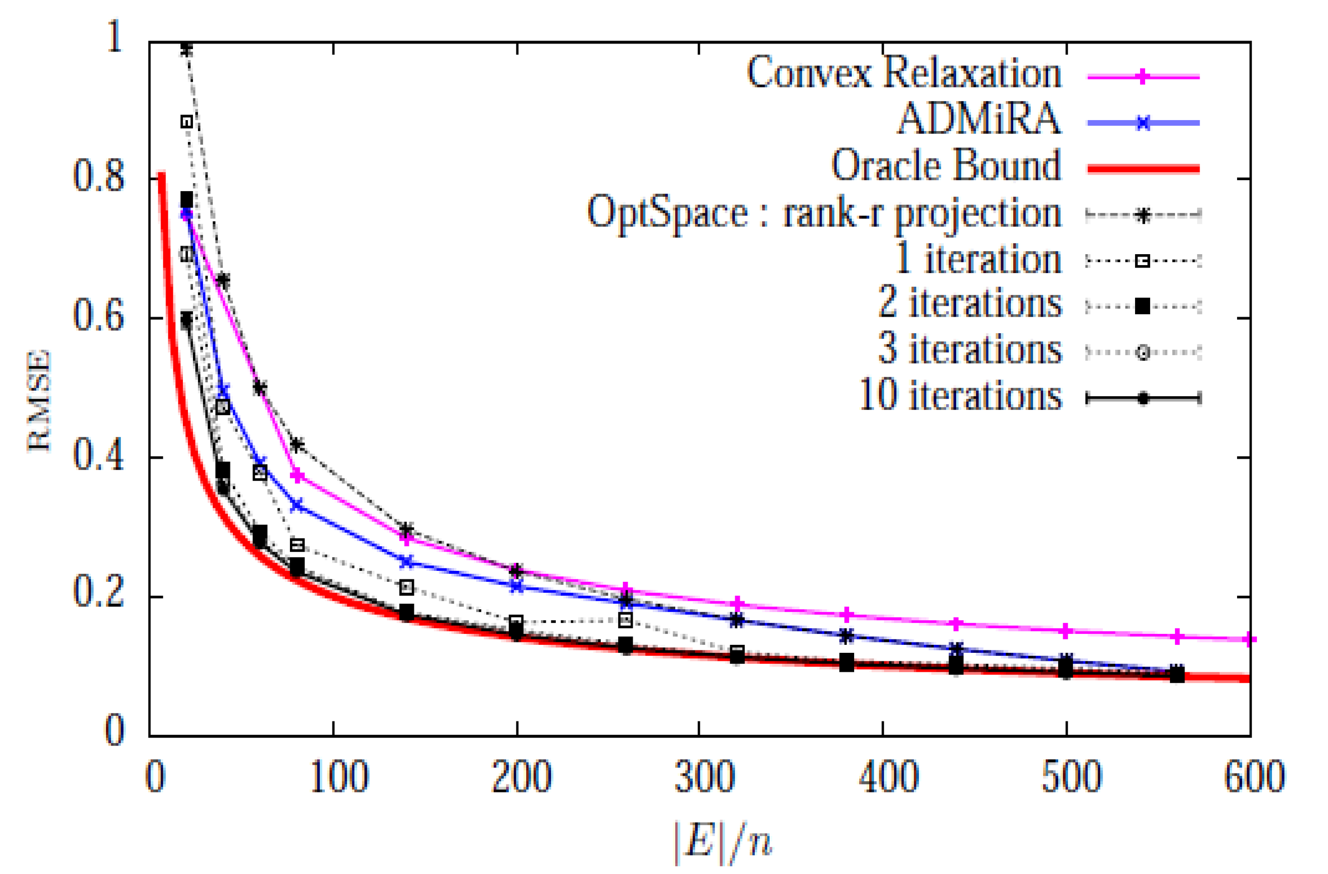


Figure 2: RMSE vs. Number of samples revealed (m = n = 600, r = 2)

References

[1] R.H.Keshavan, A. Montanari, and S. Oh, *Matrix Completion from a few entries* , arXiv:0901.3150, January 2009.

[2] R.H.Keshavan, A. Montanari, and S. Oh, *Matrix Completion from noisy entries* , arXiv:0906.2027, June 2009.