Fast Graph Laplacian Regularized Kernel Learning via Semidefinite–Quadratic–Linear Programming

Xiao-Ming Wu, Anthony Man-Cho So, Zhenguo Li, and Shuo-Yen Robert Li
The Chinese University of Hong Kong

Laplacian Regularized Kernel Learning
Quadratic SDP

$$\min_{\mathbf{y}} \ \mathbf{y}^T A\mathbf{y} + \mathbf{b}^T\mathbf{y}$$
$$\text{s.t. } Y \succeq 0$$

Previous approaches turn the quadratic term to a positive semi-definite constraint

$$\min_{\mathbf{y},\nu} \ \nu + \mathbf{b}^T\mathbf{y}$$
$$\text{s.t. } Y \succeq 0 \text{ and } \begin{pmatrix} I_{m^2} & A^{\frac{1}{2}}\mathbf{y} \\ (A^{\frac{1}{2}}\mathbf{y})^T & \nu \end{pmatrix} \succeq 0$$

Complexity is of the order m^9
(m is the dimension of Y)

Our formulation turn the quadratic term to a second order cone and a set of linear constraints

$$\min_{\mathbf{y},\mathbf{u}} \ (\mathbf{e}_1 - \mathbf{e}_2)^T\mathbf{u} + \mathbf{b}^T\mathbf{y}$$
$$\text{s.t. } (\mathbf{e}_1 + \mathbf{e}_2)^T\mathbf{u} = 1,$$
$$B\mathbf{y} - C\mathbf{u} = \mathbf{0},$$
$$\mathbf{u} \in \mathcal{K}_{r+2},$$
$$Y \succeq 0,$$

Complexity is of the order $m^{6.5}$

Gain in speed up: $m^{2.5}$