

Worst-case bounds on the quality of max-product fixed-points (T75)

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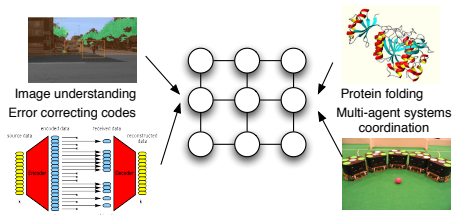
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Motivation

Many **real-world problems** can be represented using a **graphical model** and formulated as **finding the maximum a posteriori assignment (MAP)**.



Loopy max-sum belief propagation is a popular **MAP algorithm** with **good empirical performance** but with **few theoretical guarantees**.

[Open question:] can we **provide theoretical guarantees** for max-sum algorithm **over further structures** upon convergence?



We provide **quality guarantees for max-sum fixed-points** in **general graphs calculated prior to the execution** of the algorithm.

Our results build upon two main components:

- The **characterization of any max-product assignment**, x^{MS} , as a **neighbourhood maximum** in an specific region of the graphical model [\[Weiss and Freeman, 2001\]](#)



- **Worst-case bounds** on the **quality** of a **neighbourhood maximum**, generalization of [\[Pearce and Tambe, 2007\]](#).

$$\theta(x^{MS}) \geq \frac{cc_*}{|C| \cdot nc_*} \theta(x^*)$$





Number of times covered the less covered edge

Number of sets in the region

Number of times non-covered the less non-covered edge

(All bounds provided are independent of the specific MRF parameters)

$$\theta(x^{MS}) \geq \alpha \cdot \theta(x^*).$$

	Complete	(n, m) - Bipartite	2-D n^2 -Grids	Variable-disjoint cycles
α	$\frac{1}{ V -2}$	$\frac{n}{3n-4}$	$\alpha(n, m) = \begin{cases} \frac{1}{n} & m \geq n+3 \\ \frac{2}{n+m-2} & m < n+3 \end{cases}$	$\left(1 - \frac{2(d-1)}{d \cdot l}\right)$
e.g.	 $\frac{1}{2}$	 $\frac{1}{2}$	 $\frac{1}{2}$	 $\frac{3}{4}$

