

A family of penalty functions for structured sparsity

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Sparsity with Lasso and variational form

We consider a linear regression model under **structured sparsity** assumptions on β^* , the underlying vector. A popular estimate for the sparse case is

$$\hat{\beta}^{\text{LASSO}} = \underset{\beta}{\operatorname{argmin}} \left\{ \|y - X\beta\|_2^2 + \|\beta\|_1 \right\}.$$

Equivalently, the ℓ_1 norm is the solution of a **convex** problem:

$$\|\beta\|_1 = \inf_{\lambda > 0} \frac{1}{2} \sum_{i=1}^n \left(\frac{\beta_i^2}{\lambda_i} + \lambda_i \right),$$

where $\lambda \in \mathbb{R}^n$ is a vector of **auxiliary** variables.

Constrained variational form

We introduce the **additional constraint** $\lambda \in \Lambda \subseteq \mathbb{R}_{++}^n$:

$$\Omega(\beta|\Lambda) = \inf_{\lambda \in \Lambda} \frac{1}{2} \sum_{i=1}^n \left(\frac{\beta_i^2}{\lambda_i} + \lambda_i \right).$$

The set Λ provides a **structure** in the sense that the function is minimum for $|\beta|$ in the set:

$$\Omega(\beta|\Lambda) = \|\beta\|_1 \Leftrightarrow |\beta| \in \Lambda.$$

If the set Λ is convex, so is function Ω , and we have a **convex optimization problem**.

Examples of structured sparsity

Several possibilities for the set Λ has been investigated.

- ▶ The graph with incidence matrix A :

$$\Lambda = \{ \lambda \in \mathbb{R}_{++}^n : A\lambda \geq 0 \}.$$

- ▶ Using D^k , the k -th order difference operator:

$$\Lambda = \{ \lambda \in \mathbb{R}_{++}^n : D^k(\lambda) \geq 0 \}.$$

These and other sets constraint directly λ , and indirectly $|\beta|$.