

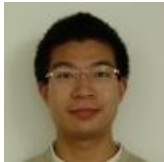
Shadow Dirichlet for Restricted Probability Modeling



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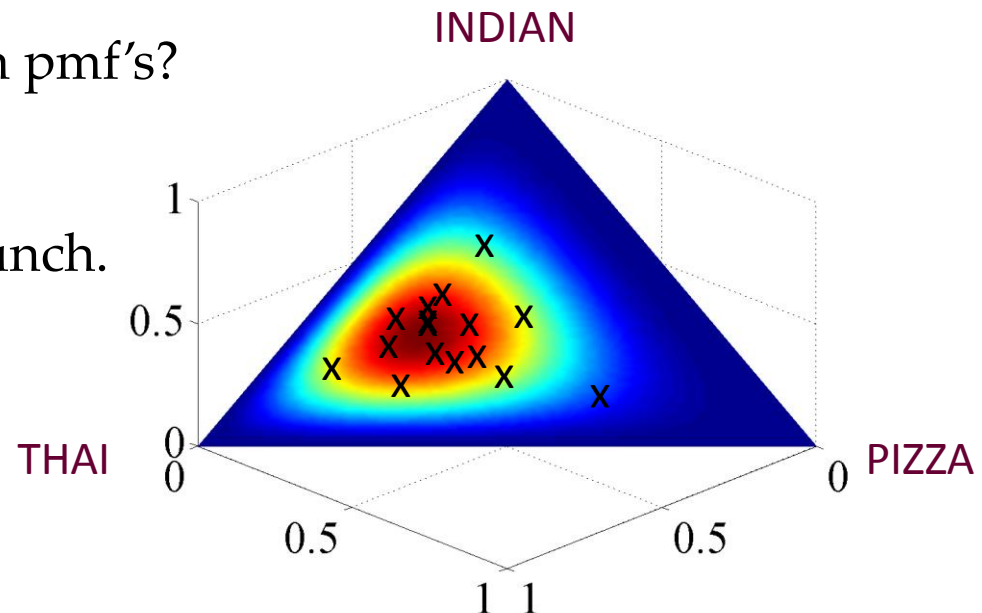
Yihua (James) Chen, Google

Problem: How do we model random pmf's?

Ex: people have different pmfs of choosing Indian, Thai, or Pizza for lunch.

Common Model:

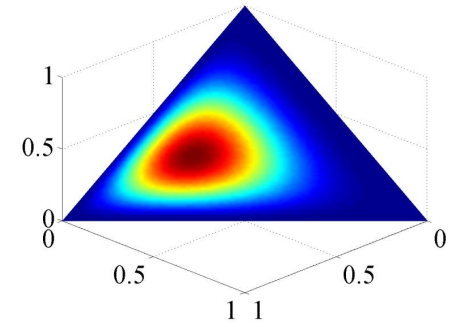
Dirichlet distribution (α),
where α is the scaled mean pmf.



Dirichlet vs Shadow Dirichlet

Dirichlet:

one parameter $\alpha \in \mathbb{R}_+^d$
every pmf possible



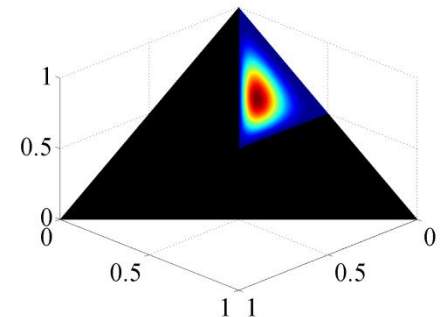
Shadow Dirichlet:

two parameters $\alpha \in \mathbb{R}_+^d$ and left-stochastic $M \in [0, 1]^{d \times d}$
new parameter M controls the support (which pmfs possible)

The random pmf $\Theta \sim \text{shadow Dirichlet}(\alpha, M)$,

$$\text{if } \Theta = M\tilde{\Theta}$$

where $\tilde{\Theta} \sim \text{Dirichlet}(\alpha)$.



Restricting the Support Can Be Used to Ensure:

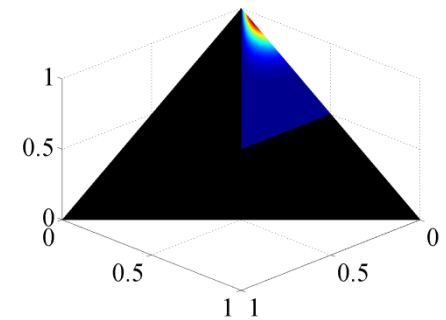
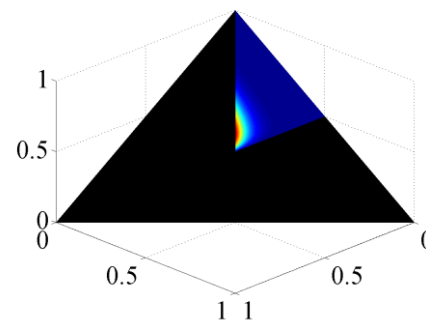
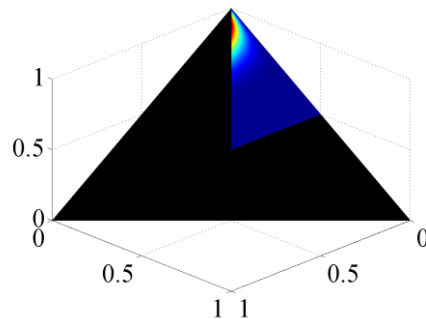
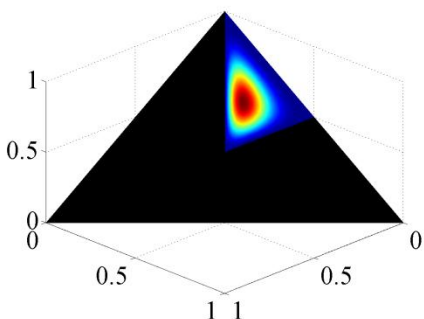
Ordered probabilities: if $\theta[i] \geq \theta[j]$

Certain Probabilities Must Be Close: if $\|\theta[i] - \theta[j]\| < \epsilon$

Regularized pmfs:

if every realization must obey $\theta = (1 - \lambda)q + \lambda q_0$ for fixed q_0

Example shadow Dirichlet distributions for $\Theta[\text{Indian}] \geq \Theta[\text{Pizza}] \geq \Theta[\text{Thai}]$



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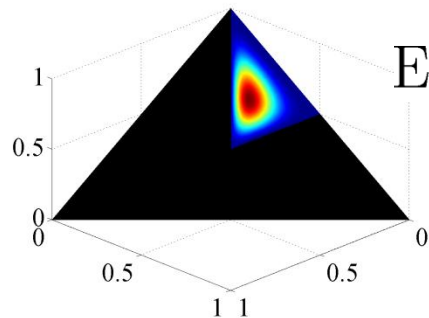
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Check out the paper/poster for:

Maximum entropy constructions of M given above constraints



EM algorithm for estimating parameters

Comparisons with Real Data

and more!