

# Trading off Mistakes and I-Don't-Know Responses in Online Learning

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# A natural theoretical question

What happens if allow "don't know" in online learning:

- World presents example (is this spam?)
- Algorithm makes prediction or says "I don't know"
- Algorithm is given correct answer.



Q: If we view saying "I don't know" as (much) better than making a mistake, how can you trade them off? [Li-Littman-Walsh: \*only\* don't knows]



Two interesting ends of spectrum:

- Just a few mistakes allowed (KWIK + a few mistakes)
- Convert as many mistakes as possible to  $O(1)$  don't knows.

Examine information-theoretically and algorithmically

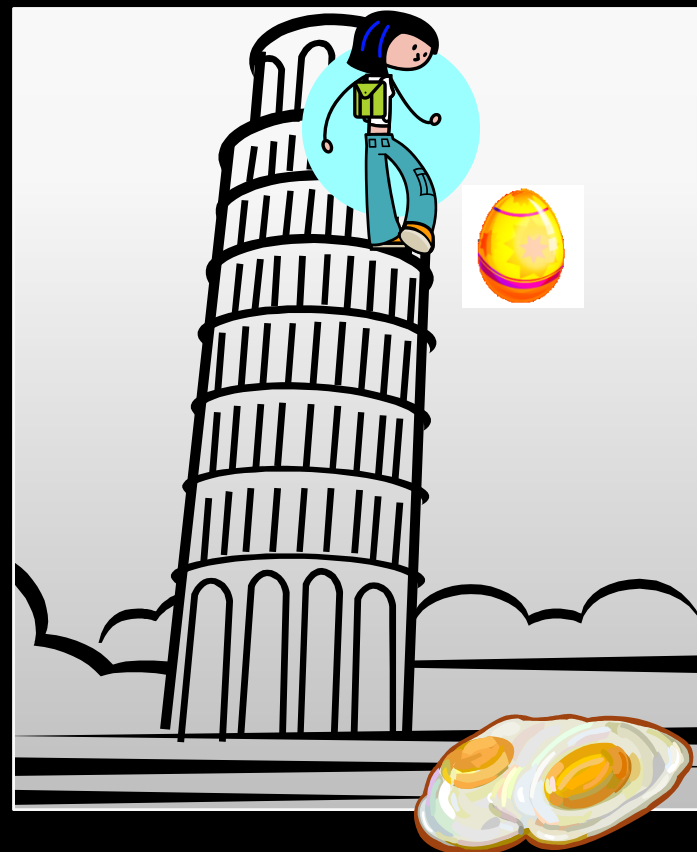
# Intuition

Information-theoretically, close connection to classic "egg dropping puzzle":

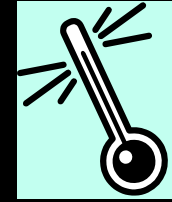
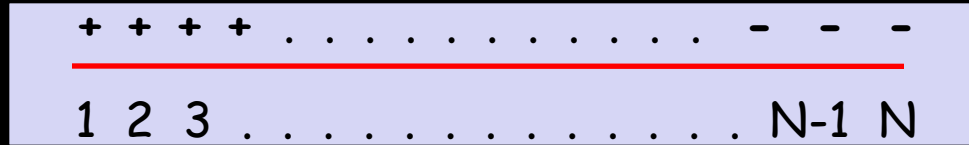
Want to figure out highest floor you can drop egg without breaking. ( $N$  floors total).

- If unlimited supply, just do binary search.
  - $O(\log N)$  drops.
- If only 1, need to do linear search.
  - $O(N)$  drops.
- What about 2 eggs?  $k$  eggs?
  - $2 \cdot \sqrt{N}$ ,  $kN^{1/k}$  drops.

Use this to show: if allow  $k-1$  mistakes, can learn any  $f \in C$  with  $k|C|^{1/k}$  don't know.



Helpful example: initial intervals of  $1, \dots, N$ .



## Algorithmic Results

Disjunctions: classic bound of  $n$  mistakes (necessary and sufficient in worst case)

- Reduce to  $n/2$  mistakes +  $O(n)$  don't knows.
- Reduce to  $n/3$  mistakes +  $O(n)$  don't knows.

### Linear separators

- Use random sampling to estimate volume of version space voting + or -.
- Bounds a bit more complex: **see poster.**